

ZM-138**May-2014****M.Sc., Sem.-II****407 : Physics****(Quantum Mechanics-II and Mathematical Physics-II)****Time : 3 Hours]****[Max. Marks : 70**

- Instructions :** (1) Numbers to the right margin indicate full marks of the respective question.
 (2) Symbols and terminology have their traditional meaning.
 (3) Use Scientific calculator, if required.

1. (a) Discuss Heisenberg picture, and show that it leads to Hamiltonian's Equation of Motion. 7

OR

What is the basic difference between Heisenberg picture and Schrodinger picture for equation of motion ? Prove that in Schrodinger picture for any dynamical variable A, $\langle A \rangle_\phi$ is a constant of motion.

- (b) What are approximations involved in the Thomas-Fermi method ? Derive an expression for electron density $n(r)$ using the Thomas-Fermi approximation. Mention limitations of the Thomas-Fermi model. 7

OR

Write Hamiltonian for many-particle system. How variational technique can be used to solve Schrödinger equation. What do you mean by self-consistent field approximation ?

2. (a) Calculate Einstein coefficient A_{21} for spontaneous emission for transition from $2P \rightarrow 1S$ state in hydrogen atom, given 7

$$|\Psi_{1S}\rangle = (\pi a_0^3)^{-1/2} \exp\left(-\frac{r}{a_0}\right) \text{ and}$$

$$|\Psi_{2P}\rangle = (\pi)^{-1/2} (2a_0)^{-3/2} x \exp\left(-\frac{r}{a_0}\right).$$

OR

Write an equation for vector potential $A(r, t)$. Derive an equation for average electric field, $\langle \epsilon_k \rangle = \frac{1}{2} (\omega_k Q_k^2 + P_k^2)$

- (b) What are coherent states ? Show that coherent states are identical to classical state. 7

OR

Discuss interaction picture of radiation with matter. Derive an expression for interaction Hamiltonian in terms of raising and lowering operators.

3. (a) Write Cauchy-Riemann's first and second conditions. Write statement and give proof of Cauchy Integral Theorem for multiply connected Regions. 7

OR

Write statement of Morera's theorem and show that $\oint_C f(z) dz = 0$.

- (b) What is importance of the Cauchy-Riemann conditions ? Show that u and v follow Laplace equation in two dimensions. 7

OR

Evaluate the integral $I = \int_0^{2\pi} \frac{\cos 3\theta}{5 - 4 \cos \theta} d\theta$.

4. (a) The general second-order linear differential equation with constant coefficients is : 7

$$y''(x) + a_1 y'(x) + a_2 y(x) = 0$$

With the boundary conditions

$$y(0) = y(1) = 0$$

Develop integral equation with appropriate Kernel.

OR

Using the Neumann series, solve

$$\phi(x) = 1 - 2 \int_0^x t \phi(t) dt$$

- (b) State the properties of one dimensional Green's function. Explain eigen function expansion of one dimensional Green's function. 7

OR

Describe Fourier transform method of constructing the Green's function.

5. Answer the followings : (Each of **one** mark)

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- (i) If $|\alpha_1\rangle$ and $|\alpha_2\rangle$ are two coherent states, then what will be the value of $\langle\alpha_1|\alpha_2\rangle$?
- (ii) If \hat{H} is a dynamical variable in Schrodinger picture, then what will be the constant of motion in state Ψ ?
- (iii) What will be time dependent Hamiltonian for dipole interactions ?
- (iv) If $\hat{F} = \hat{P}$, where \hat{P} is linear momentum operator, then how we can write uniform transformation $\hat{U}(t, t_0)$?
- (v) Find the value of product of $\hat{U}(t_1, t_2)$ with $\hat{U}(t_2, t_3)$.
- (vi) What will be the dimension of 'b' in Thomas-Fermi approximation ?
- (vii) What do you mean by spherically symmetric potential ?
- (viii) Prove that $f(z) = \bar{Z}$ is non analytic function.
- (ix) Write Taylor's series of a function $\sin Z$.
- (x) What is multiple pole ?
- (xi) What is a holomorphic function ?
- (xii) When the differential equation is said to be non-homogeneous ?
- (xiii) Write an expression for Volterra equation of the second kind.
- (xiv) When the Neumann series is convergent ?
